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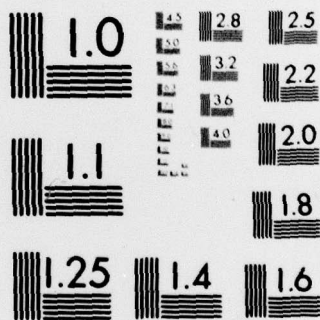   AUG 79   J C CONLON   AFOSR-78-3670
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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

One investigator was supported and conducted research under this grant. Three reports were prepared under this grant and one chapter of a major monograph was also prepared under this grant. The reports dealt with the applications of the theory of G-ordered function in reliability; the applications of the theory of G-monotone functions in reliability; and some majorization results for a system of flow. The monograph deals with the topics of partial orderings on N-dimensional space and their use in probability and statistics.

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Final Report of Progress on AFOSR 78-3670:

"A general approach to ordering of parameters, with application in reliability"

Principal Investigator: John C. Conlon

Department of Mathematical Sciences, The University of Akron

Grant Period: July 1, 1978 through June 30, 1979

Date of Report: August, 1979

Preface

This is the final report of work done on AFOSR grant 78-3670 "A general approach to ordering of parameters, with application in reliability." The grant period is July 1, 1978 through June 30, 1979. In this report we:

1. List specifically the accomplishments in terms of technical reports being prepared.

2. Give, for each technical report, a technical summary, and a non-technical summary with the latter describing potential applications to the Air Force.

1. List of Accomplishments

Technical Reports in preparation:

1. Applications of the theory of G-ordered functions in reliability.

2. Applications of the theory of G-monotone functions in reliability.

3. Some majorization results for a system of flow.

Monograph in preparation:

1. **Partial Orderings on N-dimensional space and their Use in Probability and Statistics.**

2. Discussion of Papers

2.1. Applications of the theory of G-ordered functions in reliability.

2.1.a. Abstract.

We focus on one specific type of reflection group, which we denote by the letter G. The group G is the cross product of an arbitrary number of permutation groups. We present sufficient conditions on the distribution of a random vector x under which the expected value of a G-ordered function of x is G-ordered. We demonstrate a parametric test of hypothesis which has a G-ordered power function. We also derive a permutation-type test for the same hypothesis which again has a G-ordered power function. Some sample applications are thoroughly developed.

2.1.b. Potential Use to Air Force.

In this paper we have chosen applications in reliability to show the usefulness of the theory. One application is the selecting of parts from an inventory to construct a set of systems in series. We complicate the problem by defining the actual reliability or lifelength of each component to be the product of the original reliability or lifelength and some sort of reduction factor. The reduction factors may come about as a result of the amount of time on the shelf, amount of previous use, or quality of craftsmanship and materials. The theory easily encompasses the following three cases: random reliabilities or lifelengths, random reduction factors, and both random reliabilities or lifelengths and random reduction factors.

A second application relates to a system of flow. The model for a system of flow is the flow of water through a sequence of junctures with some number of pipes between each pair of adjacent junctures. The amount of flow which passes through a single pipe is the product of the pipe's capacity and a reduction factor

due to leakage or blockage. We consider that either the capacities, the reduction factors or both may be random.

2.2. Applications of the theory of G-monotone functions in reliability.

2.2.a. Abstract.

We focus on one specific type of reflection group, which we denote by the letter G. The group G is the cross product of an arbitrary number of permutation groups. We present sufficient conditions on the distribution of a random vector x under which the expected value of a G-monotone function is G-monotone. We demonstrate a parametric test of hypothesis which has a G-monotone power function. We establish admissibility criteria for nonparametric tests of the same hypothesis. Some sample applications are thoroughly developed.

2.2.b. Potential Use to Air Force.

Here also we have chosen applications in reliability to show the potential use of this theory. We present results for both series and parallel systems, considering either the reliability or the lifelength of the system. We describe a partial ordering on assemblies of a sequence of systems. Not only do we know the optimal assembly, but we can determine if one suboptimal assembly is better than another. This approach is very useful in reliability growth problems since we know when we are improving the system even though the best system has not been achieved.

We present a test of hypothesis whereby we can determine if the reliabilities or lifelengths of components come from a distribution with equal parameters or not.

2.3. <u>Some majorization results for a system of flow</u>.

2.3.a. <u>Abstract</u>

A system of flow may be, for example, water through pipes or electricity through wires. We assume that the system is composed of a sequence of junctures at which points some characteristic of the flow changes. Variables of interest are reliabilities, lifelengths, capacities, and resistances. When the distribution of each variable comes from a family of distributions indexed by a location parameter, under certain conditions the distribution of the minimum of any finite number of these variables is a Schur-convex function. We also demonstrate conditions under which the distribution of the total amount of flow passing through the system is a Schur-convex function of the number of pipes between each pair of junctures.

2.3.b. <u>Potential Use to Air Force</u>.

Systems of flow as we have defined them occur routinely in a number of situations. The main thrusts of the results are the establishment of a partial ordering of designs and the direction of system repair. We have pointed out conditions which are sufficient for designing systems which have an optimal property. We have also indicated how violation of some condition may result in the design on an inferior system. We have shown that the amount of flow which passes through a system may be used as a statistic for testing an hypothesis about equal flow between junctures.

2.4 <u>Partial Orderings on N-dimensional space and their Use in Probability and Statistics</u>.

2.4.a. <u>Chapter 2. Majorization Orderings</u>.

We have collected all known characterizations of the original majorization ordering, as well as the weak majorization orderings. We define functions isotonic

with respect to each ordering and develop the preservation properties of these functions. A substantial list of applications of the theory in probability and statistics indicates how these orderings have been used.

2.4.b. Potential Use to Air Force.

The book is intended to be used as a reference work for those individuals who have need of this theory. We present the applications so that the reader may see how the theory has been used in the past. The potential applications are limited only by the imagination of the reader. Use to Air Force personnel would be limited strictly to those involved in statistical research.

3. Travel Activities of the principal investigator during the contract period.

Two trips to Florida State University to work with Frank Proschan (and others) in preparing the technical reports and beginning work on the monograph.